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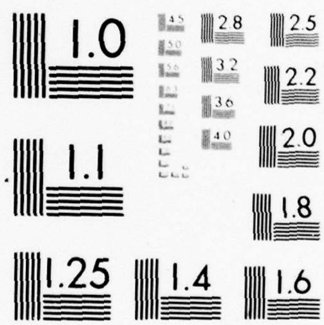

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SIMPLE FORMULAE FOR CALCULATING THE VIBRATIONAL FREQUENCIES OF PLATES, SHELLS AND MEMBRANES

by

R. JONES

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SIMPLE FORMULAE FOR CALCULATING THE VIBRATIONAL FREQUENCIES OF PLATES, SHELLS AND MEMBRANES

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SUMMARY

Simple approximate formulae for estimating the vibrational frequencies of plates, shells and membranes are presented. These formulae take the general form $F = A\sqrt{W_{max}}$ *where F is a frequency parameter, A is a numerical parameter having a constant value for many different problems, and* W_{max} *is the maximum deflection of an associated static deflection problem.* W sub max

The approach is particularly simple and the agreement between the frequencies calculated using the approximate formulae and those calculated using standard vibration analyses is excellent.

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NOTATION

ρ	Mass per unit area
h	Thickness of the plate, membrane or shell
ν	Poisson's ratio
ω	Frequency
W'_{max}	Maximum deflection due to a uniformly distributed load
$\hat{W}_{max}$	Maximum deflection due to a load varying linearly with the distance from the centroid
k	Curvature of a shallow cylindrical or spherical shell
ω_0	Frequency parameter for a flat plate with Poisson's ratio equal to zero
E	Young's Modulus
D_x, D_y, H	Flexural rigidities for an orthotropic plate
δ	Aspect ratio

1. INTRODUCTION

Static deflection modes, due to particular pressure distributions, have long been used for vibration and flutter analysis, and are commonly used in conjunction with the Ritz method to obtain estimates of the vibrational frequencies. However, in a recent paper, Jones[1] proposed a new procedure which related the fundamental frequency of a vibrating plate to the maximum deflection of the corresponding, uniformly loaded plate. Unlike the Ritz method, this technique has the advantages that it is not necessary to perform any integration, and that the approximation is explicitly independent of the support conditions.

The aim of the present report is to discuss the developments of this technique, which have enabled the method to be extended so as to predict the frequencies of several higher modes as well as the frequencies of membrane and shell structures.

2. FORMULA FOR PLATES

In a recent article, Jones[1] presented an approximate expression for the fundamental frequency of a vibrating plate: viz.

$$\rho h\omega^2 = (10\cdot2151)^2/64W'_{max} \tag{1}$$

where ρ is the mass per unit area, h is the plate thickness, ω the frequency of vibration and W'_{max} is the maximum deflection under unit loading of the corresponding uniformly loaded plate. Although the accuracy of this expression was clearly shown in Reference 1, this paper will include a more detailed investigation into its accuracy. In Table 1, the predicted values of the frequency, along with corresponding values obtained from the literature are shown for a large range of plates of various geometrical shapes and subject to a combination of boundary conditions.

TABLE 1

Frequency Parameter $\omega a^2\sqrt{\rho h/D}$ for Fundamental Mode for Various Plate Problems, $\nu = 0\cdot3$ (unless otherwise stated)

Problem	Equation (1)	Literature, e.g. Ref. 3
Simply supported equilateral triangle, sides of length $2a/\sqrt{3}$	39·80	39·48
Clamped rectangular plate, sides of length a, b		
$b/a = 1\cdot0$	35·97	35·99
$= 1\cdot5$	27·22	26·59
$= 2\cdot0$	25·33	24·58
Simply supported rectangular plate		
$b/a = 1\cdot0$	20·04	19·74
$= 1\cdot5$	14·53	14·26
$= 2\cdot0$	12·67	12·34
Clamped semicircular plate radius a	28·40	27·? (too inaccurate)

TABLE 1—continued

Problem	Equation (1)	Literature, e.g. Ref. 3
Clamped elliptic plate, semi major axis of length a, semi minor axis of length b		
$a/b = 1 \cdot 0$	10·22	10·22
$= 1 \cdot 5$	17·20	17·20
$= 2 \cdot 0$	27·74	27·74
$= 3 \cdot 0$	58·68	58·68
Clamped rhombic plate, sides of length a, an included angle of θ		
$\theta = 75°$	38·10	38·19
$= 60°$	46·03	46·17
$= 45°$	65·77	65·59
$= 30°$	122·79	121·29
Simply supported rhombic plate		
$\theta = 60°$	24·23	23·70
$= 45°$	33·31	31·90
Rectangular plate, two opposite edges simply supported, two clamped. Length of clamped sides a, simply supported sides of length b		
$b/a = 1 \cdot 0$	29·14	28·95
$= 1 \cdot 5$	17·52	17·37
$= 2 \cdot 0$	13·90	13·69
$= 3 \cdot 0$	11·82	11·36
Circular plate of radius a clamped on the boundary over angle θ, remaining boundary simply supported		($\nu = 0 \cdot 25$)
$\theta = 0°$	4·98	4·87
$= 45°$	6·01	5·87
$= 90°$	6·49	6·35
$= 135°$	7·00	6·88
$= 180°$	7·59	7·51
$= 225°$	8·31	8·23
$= 270°$	9·13	9·12
$= 315°$	9·72	9·88
$= 360°$	10·22	10·22

TABLE 1—continued

Problem	Equation (1)	Literature, e.g. Ref. 3
Annular plate free on the inner edge		
$r = b$ and either		
(a) clamped on outer edge $r = a$,		
$a/b = 1\cdot25$	93·49	85·32
$= 2\cdot00$	17·39	17·51
$= 4\cdot00$	10·36	10·86
$= 5\cdot00$	9·97	10·34
or		
(b) simply supported on outer edge		
$r = a$		
$a/b = 1\cdot25$	9·72	9·43
$= 2\cdot00$	5·117	5·04
$= 4\cdot00$	4·58	4·625
$= 5\cdot00$	4·625	4·726
Square plate, sides simply supported, free, simply supported and clamped (SS–F–SS–C)	12·01	12·69
Square plate, three sides simply supported, the other free (SS–SS–SS–F)	11·26	11·68

It is interesting to note that Equation (1) does not depend explicitly upon the geometry or the support conditions at the edges of the plate. Furthermore it should also be noted that, in order to obtain an estimate of the frequency, it is not necessary to perform any integration. For example, let us consider a square plate clamped on all of its sides. In this case the value of W'_{max} is given by Timoshenko[2] as

$$W'_{max} = 0\cdot00126\,\frac{a^4}{D} \tag{2}$$

where a is the total length of a side of the plate. Substituting for W'_{max}, as given in Equation (2), into Equation (1), yields

$$\rho h\omega^2 = \frac{D(10\cdot2151)^2}{(64a^4)\cdot00126} \tag{3}$$

and this subsequently gives

$$\omega a^2\sqrt{\frac{\rho h}{D}} = 35\cdot97 \tag{4}$$

which differs from the value of 35·99 given by Leissa,[3] by less than 0·1%.

As a second illustration, let us consider the same rectangular plate but with two opposite sides clamped and the remaining sides simply supported. In this case the value of W'_{max} given in Reference 2 is

$$W'_{max} = \frac{0\cdot00192a^4}{D}\,. \tag{5}$$

Substituting for W'_{max} in Equation (1) now gives

$$\omega^2 a\sqrt{\frac{\rho h}{D}} = 29\cdot14 \tag{6}$$

which differs by less than 1% from the value of 28·95 as given by Leissa.[3]

The two illustrative examples considered above clearly indicate the ease of aplpying the method, while Table 1 shows its remarkable accuracy.

It should be mentioned that the formula—Equation (1)—was disco ered when the author was investigating the vibration of elliptical plates, and the constant was chosen so as to give the exact value of the fundamental frequency for the case of a clamped circular plate.

The ease and accuracy of this technique prompted Johns[4] to extend the method to the calculation of the frequency parameter for the first anti-symmetric mode. In this case the approximate expression

$$\rho h\omega^2 = \frac{40}{25\sqrt{5}\,\hat{W}_{max}} \tag{7}$$

was found to hold where $\hat{W}_{max}$ is the maximum deflection of the plate due to a load distribution which is proportional to the distance from the centroid. The accuracy of this approximation can also be judged by examining Table 2.

TABLE 2

Frequency Parameter $\omega a^2\sqrt{\rho h/D}$ for the First Anti-symmetric Mode for Various Plate Problems

	Equation (7)	Literature, e.g. Ref. 3
Circular plate of radius a		
(1) clamped	21·90	21·20
(2) simply supported	14·00	13·90
Square plate sides of length a		
(1) clamped	75·30	73·40
(2) simply supported	47·60	49·30

3. FORMULA FOR MEMBRANES AND SHELLS

Following the success of Johns'[4] extension of Jones'[1] original work, Jones[5,6] subsequently extended the approximation so as to predict the fundamental frequencies of membranes and shells of arbitrary shape. The governing equation was found to take the following generalized form

$$\rho h\omega^2 = A/W_{max} \tag{8}$$

where W_{max} is the maximum value of the corresponding static deflection mode, and A is a constant dependent upon the particular problem and the mode under consideration.

For plates and shallow shells, $A = (10{\cdot}2151)^2/64$, for the fundamental mode, and $A = 40/25\sqrt{5}$ for the first antisymmetric mode, while when predicting the fundamental frequency of a membrane of arbitrary shape,

$$A = (2{\cdot}4048)^2/4. \tag{9}$$

When predicting the fundamental frequencies of membranes, plates and shells, W_{max} is the maximum deflection which the structure would experience when subjected to a uniform loading of unity. When predicting the frequency of the first antisymmetric mode, the load distribution is to be taken as proportional to the distance from the centroid.

The accuracy of Equation (7) in predicting the frequencies of plates has already been established in Tables 1 and 2. Its accuracy in predicting the fundamental frequency of membranes can be seen in Table 3 while Table 4 shows the accuracy of the method in predicting the fundamental frequency of a simply supported cylindrical shell of curvature k. Here the fundamental

frequency ω has been compared to the exact frequency in the special case for which $k = 0$ and $\nu = 0$. The accuracy of the method can also be judged from Figures 1, 2 and 3 which show plots of the fundamental frequency of a shallow spherical shell versus the non-dimensional parameter $ka^2/2h$, where h is the shell thickness, a is the radius of the base of the shell and k is the curvature

TABLE 3

Fundamental Frequency $\omega a^2\sqrt{\rho h/E}$ of Various Shaped Membranes

Problem	Equation (9)	% difference from the literature
Elliptical membrane major axis length a minor axis length b		
$b/a = 1\cdot0$	2·405	0·0
$= 1\cdot2$	2·134	0·0
$= 1\cdot5$	2·044	0·0
$= 2\cdot0$	1·792	0·0
$= 5\cdot0$	1·734	0·0
$= \infty$	1·701	0·0
Rectangular; sides of length $2a$, $2b$		
$b/a = 1\cdot0$	2·223	0·0
$= 1\cdot1$	2·117	0·2
$= 1\cdot2$	2·041	0·2
$= 1\cdot5$	1·894	0·3
$= 2\cdot0$	1·787	1·8
$= 3\cdot0$	1·716	3·7
$= 4\cdot0$	1·707	5·4
Semicircular radius a	3·848	0·3
Equilateral triangle	6·248	0·6

TABLE 4

Fundamental Frequency Parameter $\omega a^2\sqrt{\dfrac{12\rho}{E\pi^4 h^2(1 + a^2/b^2)^2}}$ for a Cylindrical Shallow Shell

ka^2/h	1		2		3		4		5	
a/b	Pred.	Exact	Pred.	Exact	Pred.	Exact	Pred.	Exact	Pred.	Exact
1	1·519	1·520	1·062	1·063	1·076	1·081	1·095	1·105	1·108	1·136
$\frac{1}{2}$	1·072	1·072	1·136	1·140	1·221	1·246	1·308	1·381	1·389	1·536
$\frac{1}{3}$	1·089	1·086	1·198	1·193	1·341	1·351	1·513	1·545	1·674	1·766
$\frac{1}{4}$	1·098	1·093	1·234	1·219	1·387	1·403	1·647	1·627	1·880	1·875

of the shell. Exact values of the frequency are shown as given by Reissner[7] as well as the corresponding Ritz approximation (also given by Reissner[7]). In calculating the fundamental frequency of the spherical shell, exact values of the maximum deflection were not available and so use was made of the approximate expression, given in Reference 4, of

$$W'_{\max} = \frac{a^4}{128D}\left\{\frac{1}{[1 + k^2a^4(1+\nu)/8h^2]} + \frac{1}{(1 + 72k^2/6h^2)\,a^4(1+\nu)^2\,[0\cdot1 + (1+\nu)/18(1-\nu)]}\right\}. \tag{10}$$

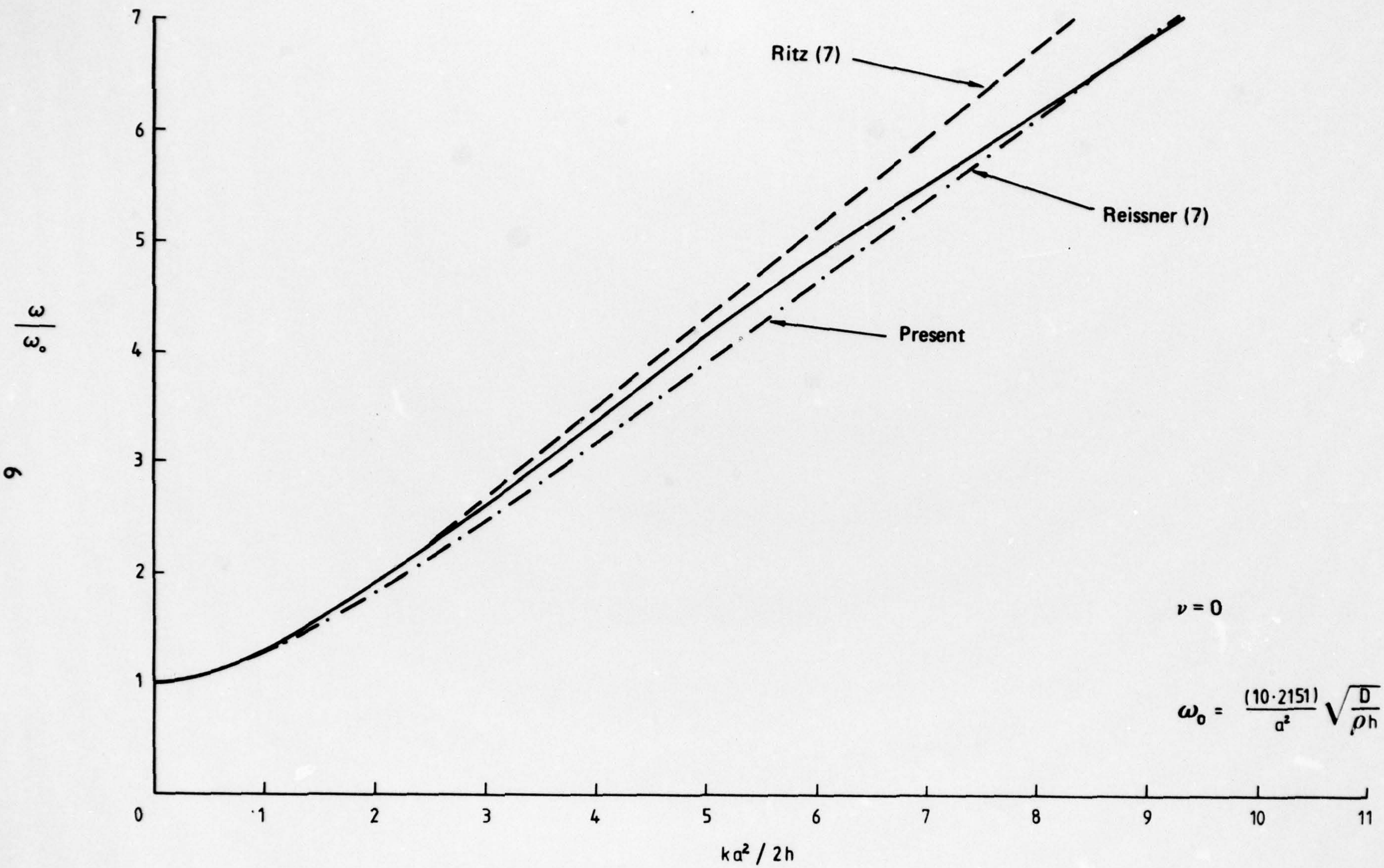

FIG. 1. FUNDAMENTAL FREQUENCY OF A SHALLOW SPHERICAL SHELL: $\nu = 0$

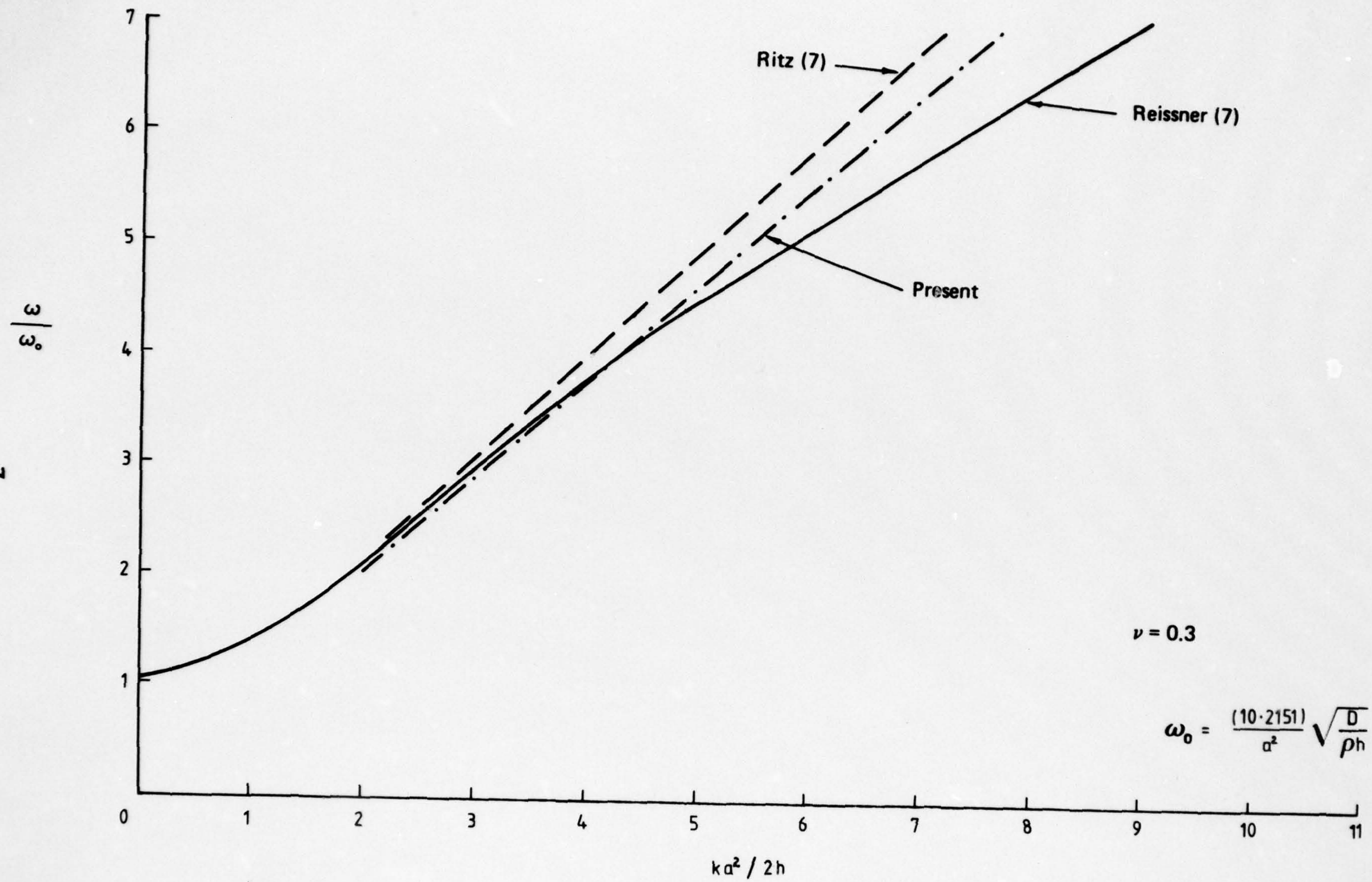

FIG. 2. FUNDAMENTAL FREQUENCY OF A SHALLOW SPHERICAL SHELL: $\nu = 0.3$

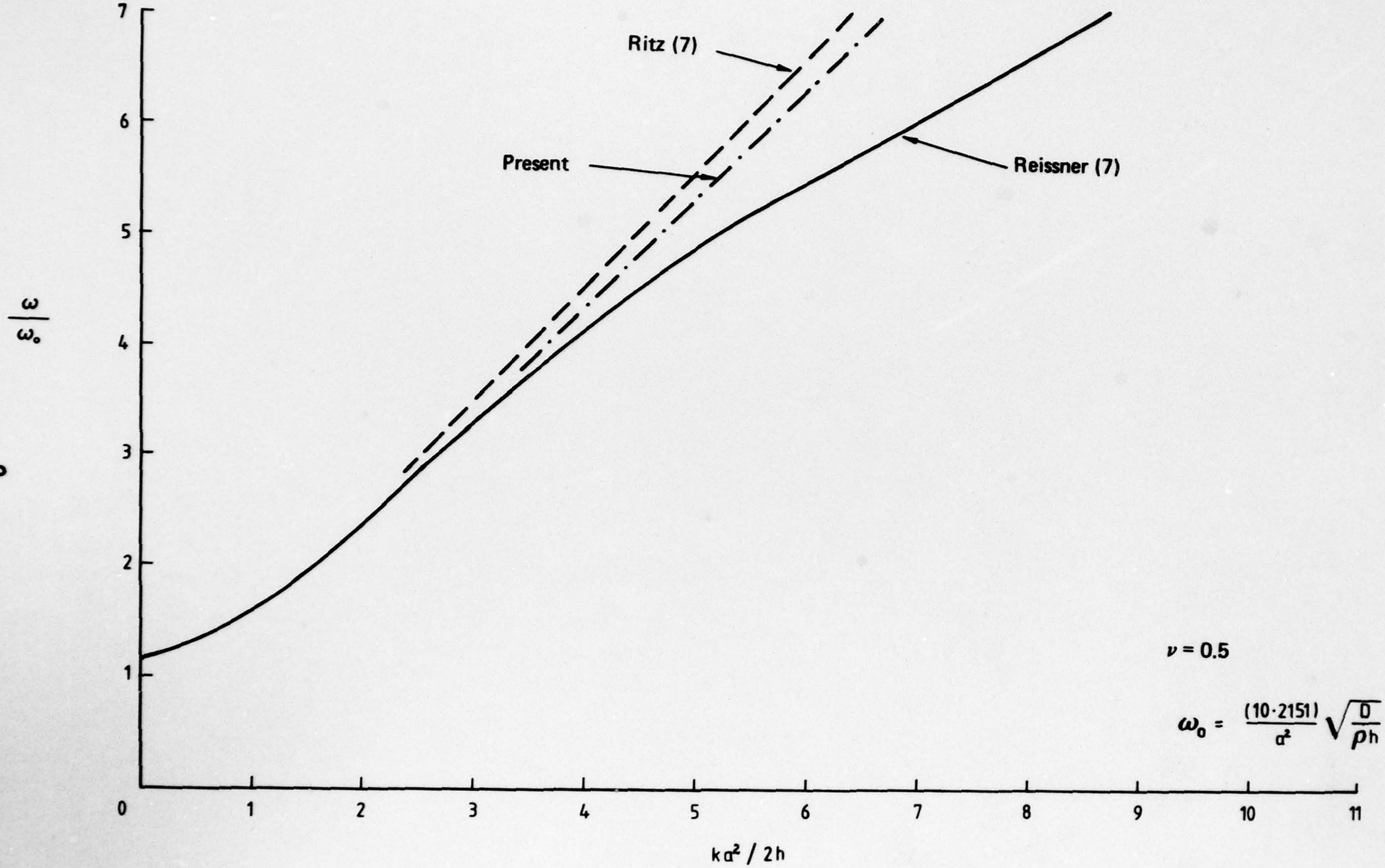

FIG. 3. FUNDAMENTAL FREQUENCY OF A SHALLOW SPHERICAL SHELL: $\nu = 0.5$

Tables 3 and 4 and Figures 1, 2 and 3 can be found in either Reference 5 or in Reference 6. We thus see that Equation (8) represents an exceptionally simple means of estimating the fundamental frequency of plates, shells and membranes.

4. FORMULAE FOR ANISOTROPIC PLATES

One very important structural form which has not yet been discussed is that of a plate consisting of orthotropic material. Let us now apply the approximate Equation (8) to the problem of a clamped elliptical plate made of composite material. In this case

$$W_{\max} = \frac{a^4}{8(3D_x + 3\delta^4 D_y + 2\delta^2 H)} \tag{11}$$

where $\delta = a/b$ is the aspect ratio of the ellipse, a, b are the lengths of the major and minor axes of the ellipse and D_x, D_y, H are the rigidities of the plate. This yields

$$\omega a^2 \sqrt{\rho h} = (10{\cdot}2151)\left(\frac{3D_x}{8} + \frac{3\delta^4 D_y}{8} + \frac{2\delta^2 H}{8}\right) \tag{12}$$

which is in exact agreement with the recent analytical results of Dharmarajan and Fang Hui Chou.[8] As a second illustration, let us consider the case of a simply supported rectangular composite plate with sides of length a and b respectively. In this case the fundamental frequency is known to be given by the relationship

$$\rho h\omega^2 = D_y\pi^4(D_x/D_y a^4 + 2H/a^2b^2D_y + 1) \tag{13}$$

while the values of $W'_{\max}$ are found in Reference 3 for the particular case $H = \sqrt{D_xD_y}$. The values of the fundamental frequency $\omega b^2\sqrt{\rho h/D_y}$ thus predicted are shown in Table 5 along with exact values given by Equation (13) above. As in all previous cases, the agreement between these two sets of values is excellent.

TABLE 5

Fundamental Frequency.

Parameter $\omega b^2\sqrt{\rho h/D_y}\backslash$ for an Orthotropic, Simply Supported Rectangular Plate; $\epsilon = a/b\sqrt[4]{D_y/D_x}$

ϵ / $\omega b^2\sqrt{\rho h/D_y}\backslash$	1·0	1·5	2·0	3·0
Predicted	20·01	14·53	12·68	11·54
Exact	19·74	14·22	12·33	10·96

We thus see that the initial work of Jones[1] has led to a new technique for estimating the fundamental frequency of common structural elements. As can be seen in the recent review article of Leissa,[9] this procedure is one of the few new techniques which may be applied to plates of arbitrary plan form with mixed support conditions.

Although we have concentrated on the positive aspects of this technique, there are two areas in which the technique is lacking.

The most important lack is the development of a rigorous mathematical basis for the approximation. So far, the only work in which this has been attempted is that of Sundarajan.[10] Unfortunately it is the opinion of the author that Reference 10 fails to provide this mathematical basis.

Again, the approximation fails in the field of large amplitude vibrations. The author has attempted to apply the approximation to the large amplitude vibration of plates but found that the predicted frequencies were in considerable disagreement with the values given in Reference 3. The method at present can only be applied when vibration amplitudes are small.

5. CONCLUSIONS

We have seen that the initial approximation developed by Jones[1] can be significantly extended so that it now provides a very simple, yet accurate, means for calculating the fundamental frequencies of membranes, plates and shallow shells vibrating at small amplitudes.

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16. *ABSTRACT*

Simple approximate formulae for estimating the vibrational frequencies of plates, shells and membranes are presented. These formulae take the general form $F = A/W_{max}$ *where F is a frequency parameter, A is a numerical parameter having a constant value for many different problems, and* W_{max} *is the maximum deflection of an associated static deflection problem.*

The approach is particularly simple and the agreement between the frequencies calculated using the approximate formulae and those calculated using standard vibration analysis is excellent.

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